Physics-Driven Deep Learning Methods for Fast Quantitative Magnetic Resonance Imaging

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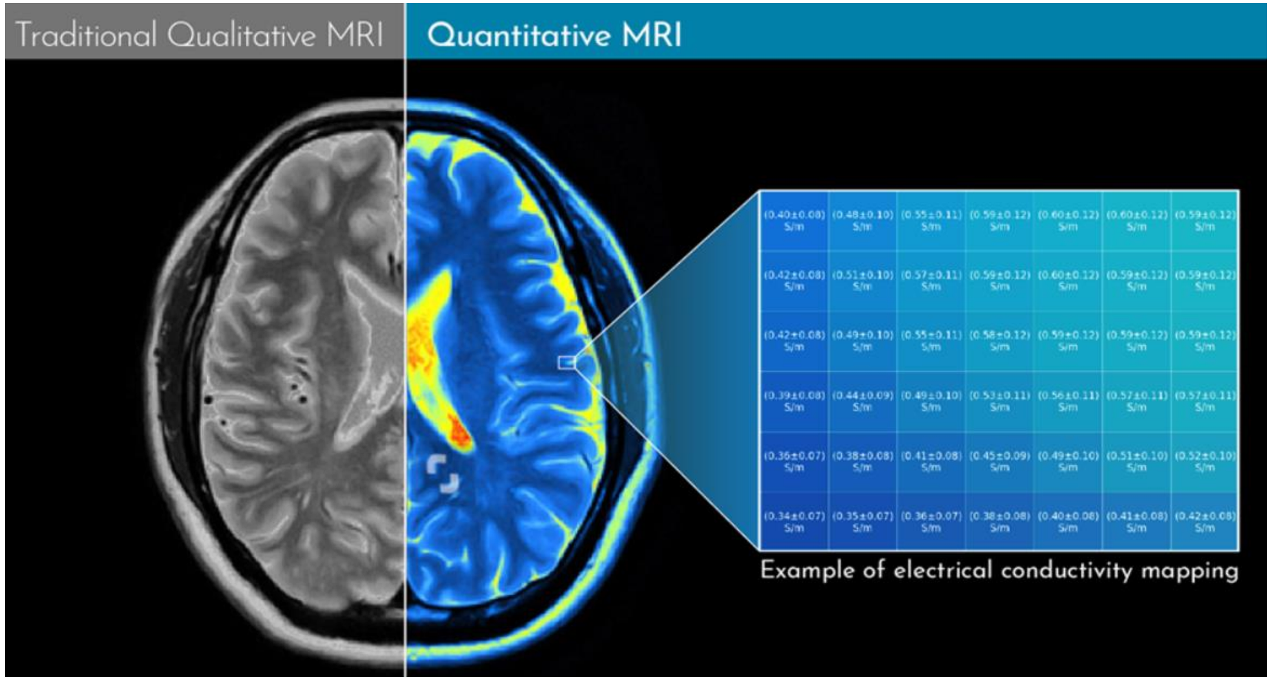
Introduction

qMRI(quantitative MRI): quantitative measurements of tissue in physical units

- Acquire contrast-weighted images
- Obtain quantitative parameter



Physics and reconstruction of qMRI

- **MR physics and physical models**

The bulk magnetization $\mathbf{M} = M_x\vec{\mathbf{i}} + M_y\vec{\mathbf{j}} + M_z\vec{\mathbf{k}}$.

When put in a static main magnetic field B_0, **Bloch equation**:

$$\frac{\mathrm{d}\mathbf{M}(\tau)}{\mathrm{d}\tau} = \mathbf{M}(\tau) \times \gamma\mathbf{B}(\tau) - \frac{M_x(\tau)\vec{\mathbf{i}} + M_y(\tau)\vec{\mathbf{j}}}{T_2} - \frac{(M_z(\tau) - M_0)\vec{\mathbf{k}}}{T_1}$$

γ : gyromagnetic ratio

M_0 : equilibrium magnetization

$\mathbf{B}(\tau)$: spatially and time-varying total magnetic field

T_1, T_2 : longitudinal and transverse relaxation time

Physics and reconstruction of qMRI

- **MR physics and physical models**

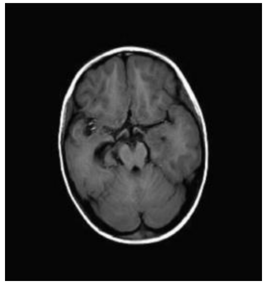
Bloch simulations describe the relationships between magnetization and biophysical parameters:

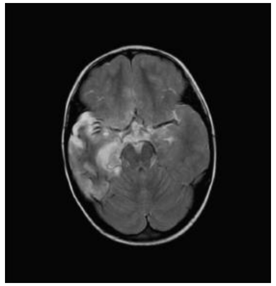
$$\rho_{t_i} = \mathbf{A}(x, t_i)$$

x : biophysical parameter

ρ_{t_i} : magnetization signal measured with the sequence parameter t_i

An MR sequence can be manipulated to "weight" the magnetization by changing a certain sequence parameter t_i to estimate the parameter x.





Physics and reconstruction of qMRI

Table 1. The commonly used physical models and their dependencies on MRI sequences.

Parameter	Model	Sequence	Varying Sequence Parameter (t_i)
Relaxation T_1	$\rho_{t_i} = I_0 \cdot \sin(t_i) \cdot \frac{1-E_1}{1-E_1\cos(t_i)}$ $E_1 = e^{-\frac{TR}{T_1}}$	Gradient echo	Flip angle
Relaxation T_2	$\rho_{t_i} = I_0 \cdot e^{-\frac{t_i}{T_2}}$	Spin echo	Echo time
Relaxation T_2^*	$\rho_{t_i} = I_0 \cdot e^{-\frac{t_i}{T_2^*}}$	Gradient echo	Echo time
Diffusion tensor $\mathbf{D}$	$\rho_{t_i} = I_0 \cdot e^{-b\mathbf{g}_{t_i}^T \mathbf{D} \mathbf{g}_{t_i}}$	Spin echo/ gradient echo	Diffusion gradient direction
Susceptibility χ	$\rho_{t_i} = \mathbb{D}(\mathbf{O}_{t_i})\chi$	Gradient echo	Orientation

Physics and reconstruction of qMRI

- **MR imaging model**

To form an MR image, the magnetization of the imaging region should be spatially encoded during data acquisition.

$$b_{t_i}^{\ell}(\mathbf{k}) = \int c^{\ell}(\mathbf{r}) \cdot \rho_{t_i}(\mathbf{r}) \cdot e^{-j2\pi\mathbf{k}^{\mathrm{T}}\mathbf{r}} d\mathbf{r} + \varepsilon_{t_i}(\mathbf{k}), \ell = 1, \ldots, \mathcal{L}$$

$b_{t_i}^{\ell}(\mathbf{k})$: measured k-space data from the l th receiver coil
$\varepsilon_{t_i}(\mathbf{k})$: associated measurement noise
$\rho_{t_i}(\mathbf{r})$: image intensity that reflects magnetization distribution in the field of view
$c^{\ell}(\mathbf{r})$: l th coil sensitivity

$\mathbf{k} \in [-0.5, 0.5)^d$ and $\mathbf{r} \in \mathbb{R}^d$ denote the k-space and image domain coordinates

$$\mathbf{b}_{t_i}^{\ell} = \mathbf{F}\mathbf{C}^{\ell}\boldsymbol{\rho}_{t_i} + \boldsymbol{\varepsilon}_{t_i} \qquad \mathbf{b}_{t_i} = \mathbf{E}\boldsymbol{\rho}_{t_i} + \boldsymbol{\varepsilon}_{t_i}$$

Physics and reconstruction of qMRI

- **Reconstruction**

Approaches of qMRI: (1) MR image reconstruction; (2) parameter fitting

- Nonlinear inverse problems:

biophysical parameters(unknowns), contrast-weighted images(observations)

$$\hat{\mathbf{x}} = \underset{\mathbf{x} \in \mathbb{C}^{N \times N_p}}{\operatorname{argmin}} \frac{1}{2} \sum_{i=1}^{L} \| \mathbf{A}(\mathbf{x}, t_i) - \boldsymbol{\rho}_{t_i} \|_2^2 \triangleq \frac{1}{2} \| \mathbf{A}(\mathbf{x}, t) - \boldsymbol{\rho}_t \|_F^2$$

x: vectorized biophysical parameter map
N_p: the number of biophysical parameters
L: the total number of images; $\mathbf{A}(\mathbf{x},t)=[\mathbf{A}(\mathbf{x},t_1),\mathbf{A}(\mathbf{x},t_2),\ldots,\mathbf{A}(\mathbf{x},t_L)]$, $\boldsymbol{\rho}_t=[\boldsymbol{\rho}_{t_1},\boldsymbol{\rho}_{t_2},\ldots,\boldsymbol{\rho}_{t_L}]$

Physics and reconstruction of qMRI

- **Reconstruction**

- Heteroscedasticity of this noise process: weighted-least-squares
- Special applications with ill-posed models: $\hat{\mathbf{x}} = \underset{\mathbf{x} \in \mathbb{C}^{N \times N_p}}{\operatorname{argmin}} \frac{1}{2} \| \mathbf{A}(\mathbf{x}, t) - \boldsymbol{\rho}_t \|_F^2 + \mathcal{R}(\mathbf{x})$
- Fast qMRI, which undersamples the k-space:

$$\hat{\mathbf{x}} = \underset{\mathbf{x} \in \mathbb{C}^{N \times N_p}}{\operatorname{argmin}} \frac{1}{2} \| \mathbf{E}\mathbf{A}(\mathbf{x}, t) - \mathbf{B}_t \|_F^2 + \mathcal{R}(\mathbf{x}; \boldsymbol{\rho}_t)$$

$\mathbf{B}_t = [\mathbf{b}_{t_1}, \mathbf{b}_{t_2}, \ldots, \mathbf{b}_{t_L}]$: acquired k-space data of all contrast-weighted images
$\mathbf{E}$: the forward imaging operator

Categories of physics-driven DL-based fast qMRI methods

Table 2. A summary of the strategies of DL-based fast qMRI methods.

Category	Advantages	Limitations	Applications
Type 1 Training sample generation via physical models	Easily implemented and low cost	The diversity of generated training data from real testing data may introduce uncertain errors.	T_2 T_1 and T_2-MRF Magnetization transfer contrast (MTC)-MRF
Type 2 Predicting missing/optimized contrast-weighted images via physical models and networks	More stable estimation from fewer contrast images	Synthesized images may be different from ideal images, and error will be propagated to quantitative maps.	MT Diffusion tensor imaging CEST
Type 3 Loss function design using synthetic *k*-space data/images generated from physical models	Adds an additional loss term, even allowing unsupervised learning	It is difficult to choose weights for different loss terms.	T_2 MTC-MRF
Type 4 Network design using physics priors as a data consistency layer	Explicitly incorporates physical models into a network and typically requires less training data	Theoretical convergence is not guaranteed.	QSM T_1

Training sample generation using physical models

- Generating training samples using Bloch simulation with a predefined parameter range.

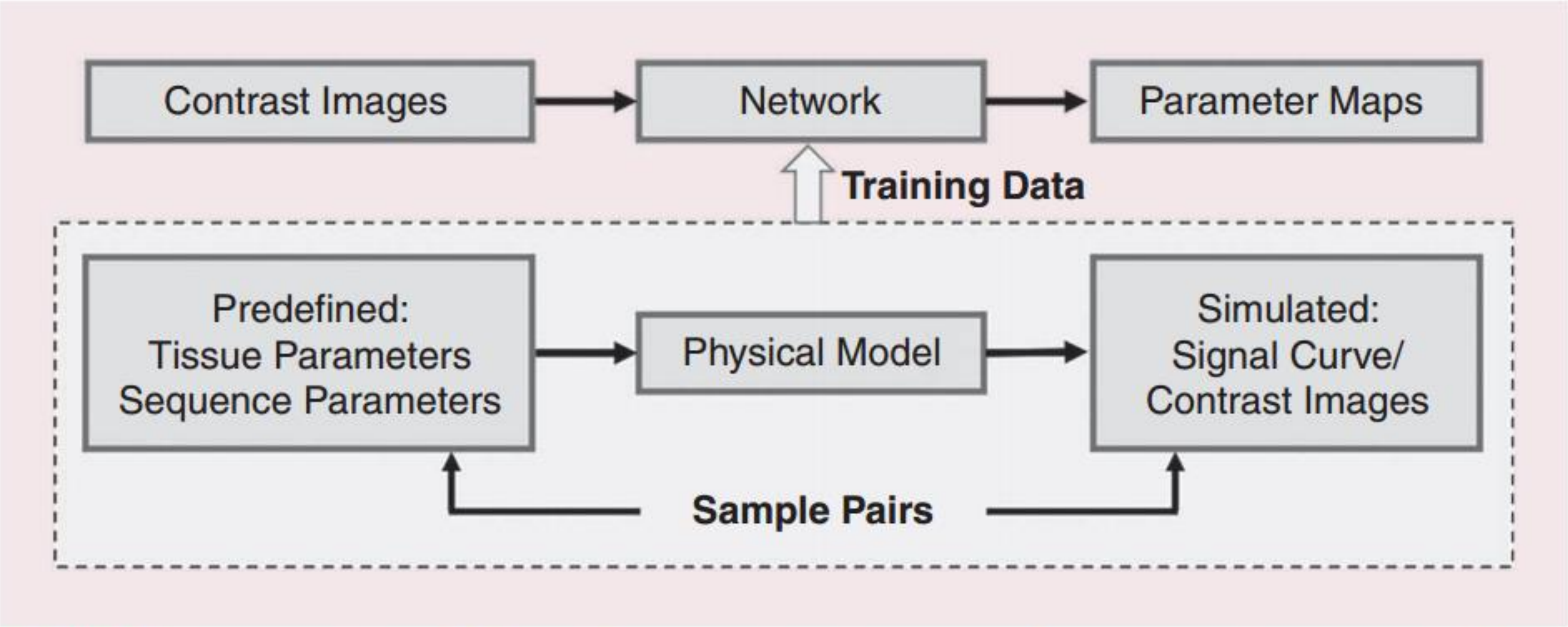


FIGURE 1. The type 1 category in Table 2. A deep network is trained for the parameter fitting of qMRI with training samples generated using a physical model.

Physical model-based synthetic images via DL

- predict missing images by using a network for replenishment
- remove disturbances by using a network for correction and improving overall image quality

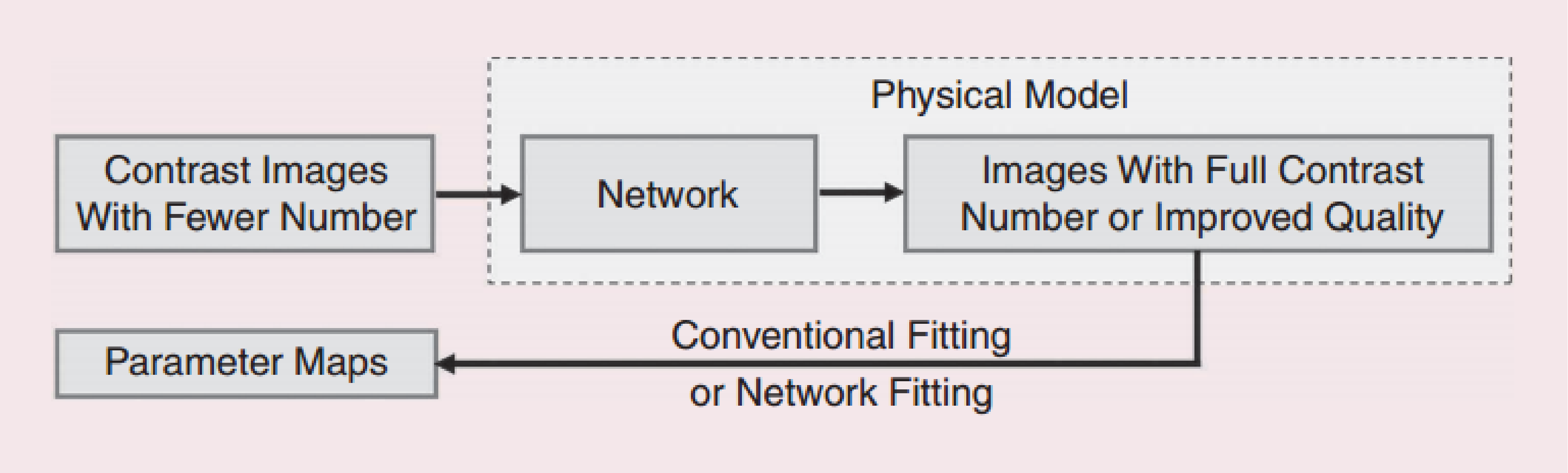


FIGURE 2. The type 2 category in Table 2. Physical models are implicitly involved when using the network to generate missing and improved images.

Physical model-based synthetic images via DL —— qMTNet

- qMT(quantitative magnetic transfer):

acquire images with multiple off-resonance frequencies for parameter fitting, typically 12 off-resonance images are acquired.

- qMTNet:

produce 2D qMT parameter maps from 4 off-resonance images by generating missing images.

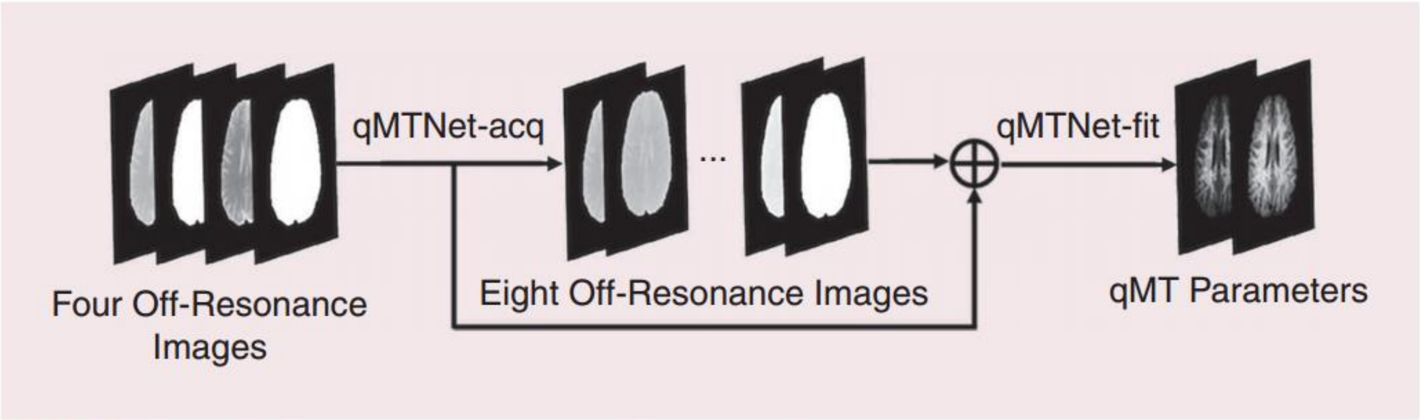


FIGURE 3. The qMTNet. Two subnetworks are involved. qMTNet-acq produces the missing eight off-resonance images from the four acquired images. qMTNet-fit obtains qMT parameters from a total of 12 images.

Physical model-integrated loss function design

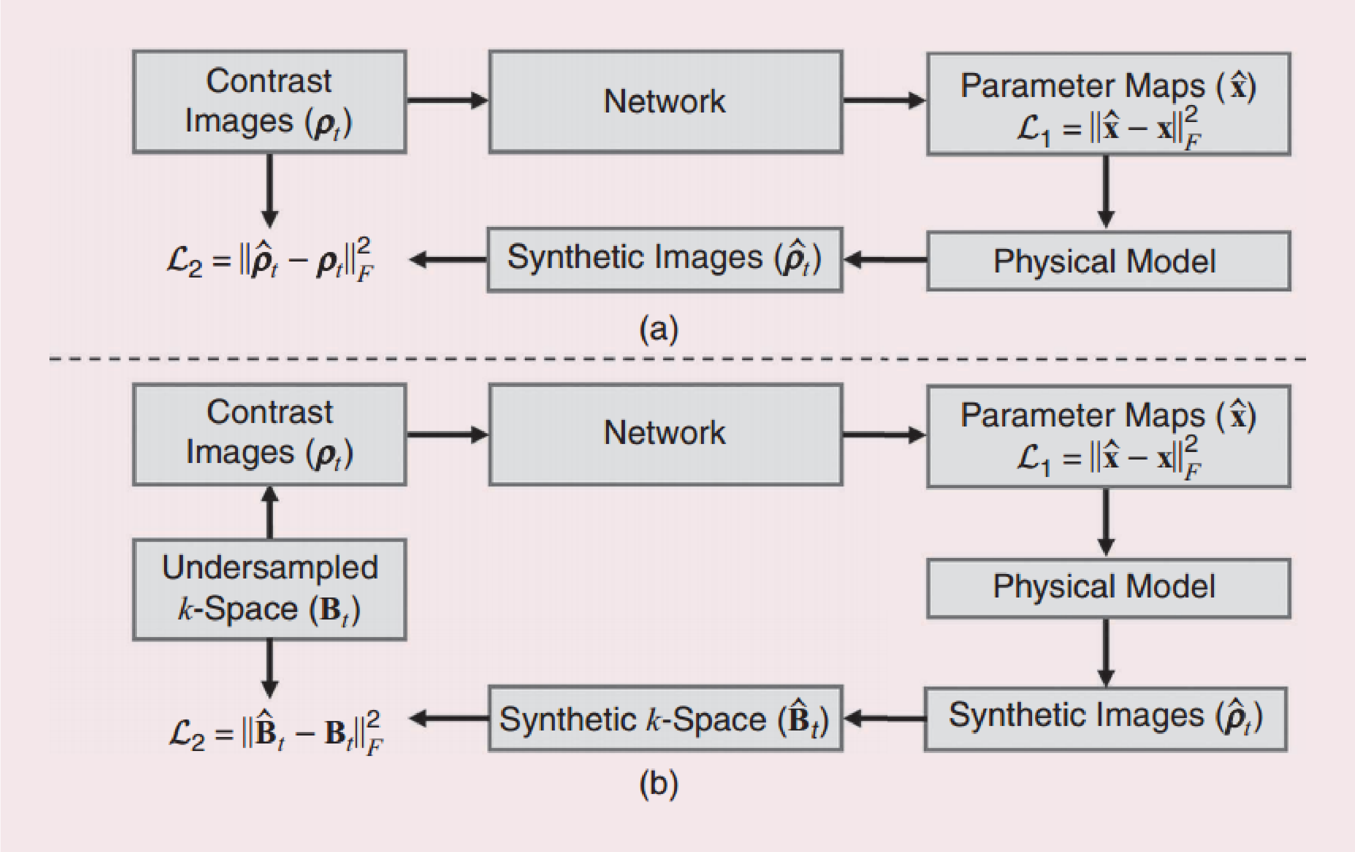


FIGURE 4. The physical model-integrated loss function design with synthetic data in the (a) image domain and (b) k- space domain. The network is used to generate parameter maps $\hat{\mathbf{x}}$ from input contrast images ρ_t. The conventional loss is the error between $\hat{\mathbf{x}}$ and the labels. The synthetic images $\hat{\rho}_t$ and k-space $\hat{\mathbf{B}}_t$ can be generated from $\hat{\mathbf{x}}$ according to the physical model and image operator $\mathbf{E}$. The error between the synthetic data and labeled data is used as an additional loss term.

$$\begin{cases} \mathcal{L} = \lambda_1 \mathcal{L}_1 + \lambda_2 \mathcal{L}_2 \\ \mathcal{L}_1 = \left\| \hat{\mathbf{x}} - \mathbf{x} \right\|_F^2 \\ \mathcal{L}_2 = \left\| \hat{\rho}_t - \rho_t \right\|_F^2 \text{ or } \left\| \hat{\mathbf{B}}_t - \mathbf{B}_t \right\|_F^2 \end{cases}$$

Physical model-integrated loss function design —— T_2 mapping

Parameter	Model	Sequence	Varying Sequence Parameter (t_i)
Relaxation T_2	$\rho_{t_i} = I_0 \cdot e^{-\frac{t_i}{T_2}}$	Spin echo	Echo time

- MANTIS (model-augmented NN with incoherent k-space sampling):

Directly estimate T_2 maps from undersampled k-space data, using a CNN with the loss function $$\mathcal{L} = \lambda_1 \left\| (\hat{\mathbf{I}}_0, \hat{\mathbf{T}}_2) - (\mathbf{I}_0, \mathbf{T}_2) \right\|_F^2 + \lambda_2 \left\| \hat{\mathbf{B}}_t - \mathbf{B}_t \right\|_F^2$$

$\mathbf{I}_0$ and $\mathbf{T}_2$: the proton density and T_2 maps to be reconstructed

$\hat{\mathbf{B}}_t = \mathbf{EA}((\hat{\mathbf{I}}_0, \hat{\mathbf{T}}_2), \mathbf{TE})$: the synthetic k-space data from the forward physical model

$\mathbf{TE}$: the echo times of T_2 mapping

- RELAX (reference-free latent map extraction):

$$\mathcal{L} = \left\| \hat{\mathbf{B}}_t - \mathbf{B}_t \right\|_F^2 + \lambda \mathcal{R}(\hat{\mathbf{I}}_0, \hat{\mathbf{T}}_2)$$

Physical model consistency network design

- With DL, iterations can be unrolled into a deep network to learn regularization and data fidelity terms.

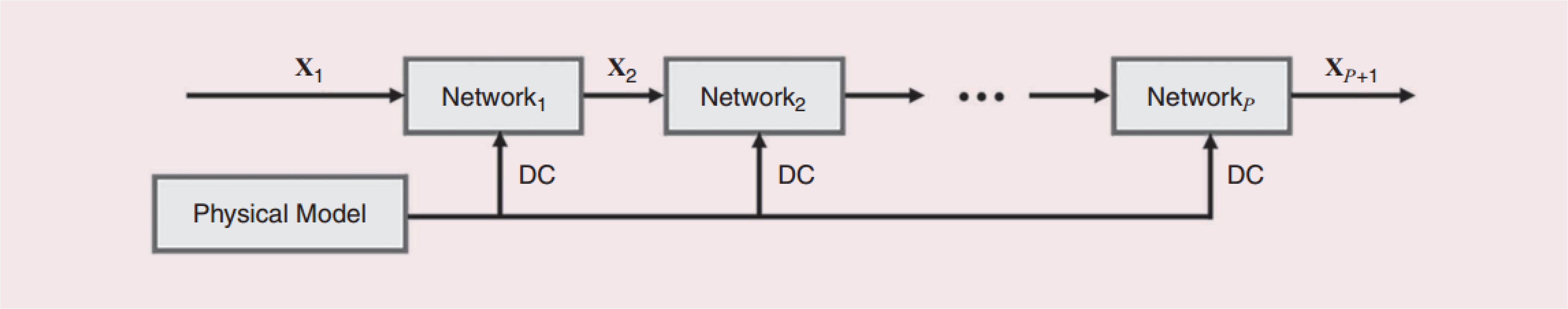


FIGURE 5. The unrolling-based network design for qMRI. Each iteration in traditional iterative reconstruction is unrolled as a network module. The physical model is incorporated as a part of the measurement operator that transfers parameter maps into the raw data space to enforce DC.

Physical model consistency network design —— T1 mapping

Parameter	Model	Sequence	Varying Sequence Parameter (t_i)
Relaxation T_1	$\rho_{t_i} = I_0 \cdot \sin(t_i) \cdot \frac{1-E_1}{1-E_1\cos(t_i)}$ $E_1 = e^{-\frac{TR}{T_1}}$	Gradient echo	Flip angle

- DOPAMINE: A deep model-based MR parameter mapping network

$$\mathcal{R}(\mathbf{X}) = \|\mathbf{X} - \mathcal{D}_{\mathrm{R}}(\mathbf{X})\|_2^2$$

where $\mathcal{D}_{\mathrm{R}}(\mathbf{X}) : \mathbb{C}^{N\times 1} \mapsto \mathbb{C}^{N\times 1}$ denotes a CNN denoiser.

$$\hat{\mathbf{X}} = \underset{\mathbf{X}}{\operatorname{argmin}} \frac{1}{2}\|\mathbf{A}(\mathbf{X}) - \mathbf{B}_t\|_F^2 + \lambda\|\mathbf{X} - \mathcal{D}_{\mathrm{R}}(\mathbf{X})\|_2^2$$

$$\begin{aligned}\mathbf{X}_{k+1} &= \mathbf{X}_k - 2\mu_k[\mathbf{J}_{\mathrm{A}}^{\mathrm{H}}(\mathbf{X}_k)(\mathbf{A}(\mathbf{X}_k) - \mathbf{B}_t) + \lambda_k(\mathbf{X}_k - \mathcal{D}_{\mathrm{R}}(\mathbf{X}_k))] \\ &= (1 - 2\lambda_k\mu_k)\mathbf{X}_k - 2\lambda_k\mu_k\mathcal{D}_{\mathrm{R}}(\mathbf{X}_k) - 2\mu_k\mathbf{J}_{\mathrm{A}}^{\mathrm{H}}(\mathbf{X}_k)(\mathbf{A}(\mathbf{X}_k) - \mathbf{B}_t)\end{aligned}$$

Physical model consistency network design —— T_1 mapping

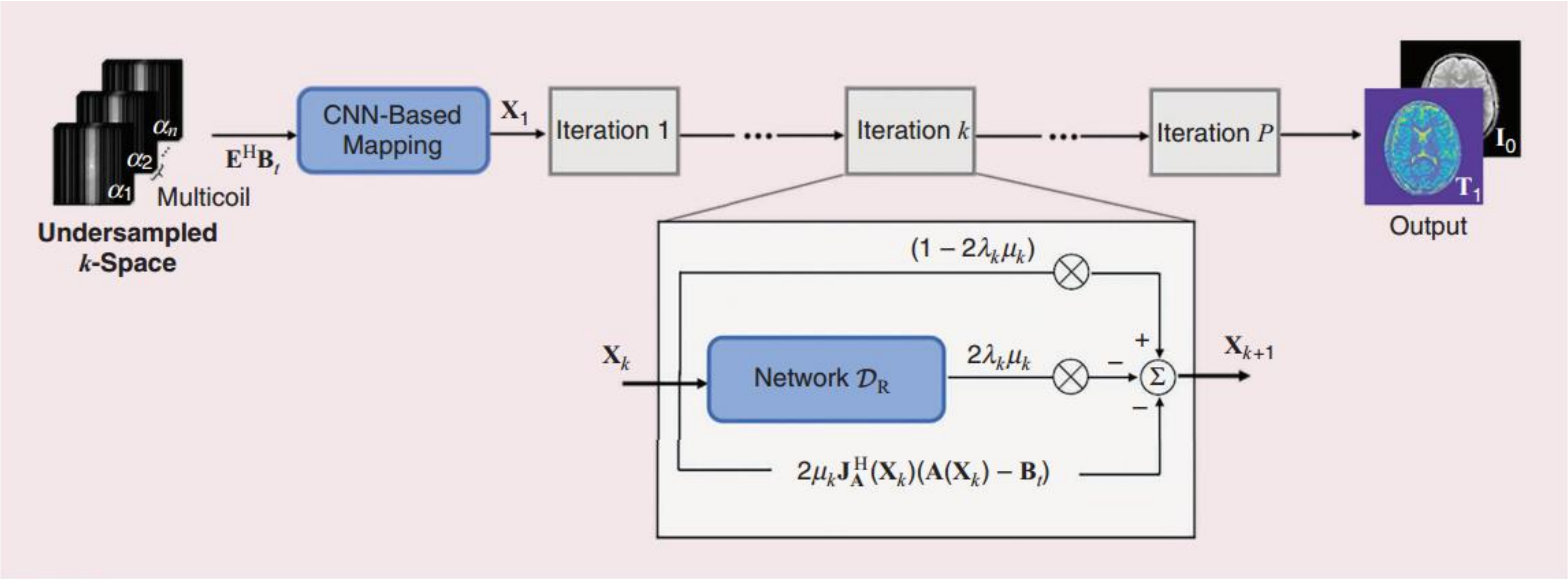


FIGURE 8. The overall architecture of DOPAMINE. The initial $\mathbf{X}_1$ is generated by the mapping network from zero-filling T_1 -weighted images. The network $\mathcal{D}_{\mathrm{R}}$ serves as a CNN-based denoiser, and the physical model is incorporated into the operation of $\mathbf{J}_{\mathrm{A}}^{\mathrm{H}}$ in the DC layer.